PMT

Model Answer
Dr. Ramez A. Elia

Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Core Mathematics C12

Advanced Subsidiary

Wednesday 24 May 2017 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. An arithmetic sequence has first term 6 and common difference 10

Find

(a) the 15th term of the sequence, **(2)**

(b) the sum of the first 20 terms of the sequence. **(2)**

$a = 6 \qquad d = 10$

(a) $U_{15} = a + 14d$

$U_{15} = 6 + 14(10)$

$\boxed{U_{15} = 146}$

(b) $S_{20} = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2(6) + 19(10)]$

$\boxed{S_{20} = 2020}$

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2. Simplify the following expressions fully.

(a) $\left(\frac{1}{9}x^4\right)^{0.5}$ **(1)**

(b) $\left(\frac{x}{\sqrt{2}}\right)^{-2}$ **(1)**

(c) $x\sqrt{3} \div \sqrt{\frac{48}{x^4}}$ **(2)**

(a) $\left(\frac{1}{9}\right)^{0.5}(x^4)^{0.5} = \frac{1}{3}x^2$

(b) $\left(\frac{\sqrt{2}}{x}\right)^2 = \frac{(\sqrt{2})^2}{x^2} = \frac{2}{x^2}$

(c) $x\sqrt{3} \times \sqrt{\frac{x^4}{48}}$

$x\cancel{\sqrt{3}} \times \frac{x^2}{4\cancel{\sqrt{3}}} = \frac{x^3}{4}$

$2 | 48$
$2 | 24$
$2 | 12$
$2 | 6$
$3 | 3$
1

$\sqrt{48} = 4\sqrt{3}$

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3. The line l_1 has equation $2x + 3y = 6$

The line l_2 is parallel to the line l_1 and passes through the point $(3, -5)$.

Find the equation for the line l_2 in the form $y = mx + c$, where m and c are constants.

(4)

$2x + 3y = 6$

$3y = -2x + 6$

$y = \frac{-2}{3}x + 2$ $\quad m = -\frac{2}{3}$

$y = mx + c$

$y = \frac{-2}{3}x + c$ $\quad (3, -5)$

$-5 = \frac{-2}{3}(3) + c$

$c = -3$

$\boxed{y = \frac{-2}{3}x - 3}$

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4. The curve C has equation $y = 4x\sqrt{x} + \dfrac{48}{\sqrt{x}} - \sqrt{8}, \quad x > 0$

(a) Find, simplifying each term,

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(5)

(b) Use part (a) to find the exact coordinates of the stationary point of C.

(5)

(c) Determine whether the stationary point of C is a maximum or minimum, giving a reason for your answer.

(2)

(a) $y = 4x^{\frac{3}{2}} + 48x^{-\frac{1}{2}} - \sqrt{8}$

(i) $\dfrac{dy}{dx} = 6x^{\frac{1}{2}} - 24x^{-\frac{3}{2}}$

(ii) $\dfrac{d^2y}{dx^2} = 3x^{-\frac{1}{2}} + 36x^{-\frac{5}{2}}$

(b) $6x^{\frac{1}{2}} - 24x^{-\frac{3}{2}} = 0$

$6\sqrt{x} - \dfrac{24}{x\sqrt{x}} = 0 \quad (\times x\sqrt{x})$

$6x^2 - 24 = 0$

$x^2 = 4$

$x = 2 \qquad x = -2 \rightarrow$ Rejected since $x > 0$

$y = 4(2)\sqrt{2} + \dfrac{48}{\sqrt{2}} - \sqrt{8}$

$y = 30\sqrt{2}$

Stationary Point $(2, 30\sqrt{2})$

(c) At $x = 2$ $\dfrac{d^2y}{dx^2} = 3(2)^{-\frac{1}{2}} + 36(2)^{-\frac{5}{2}} = 8.49$

$\because \dfrac{d^2y}{dx^2}$ is +ve $\quad \therefore$ The stationary point is minimum

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5. $$f(x) = -4x^3 + 16x^2 - 13x + 3$$

(a) Use the remainder theorem to find the remainder when f(x) is divided by $(x - 1)$. **(2)**

(b) Use the factor theorem to show that $(x - 3)$ is a factor of f(x). **(2)**

(c) Hence fully factorise f(x). **(4)**

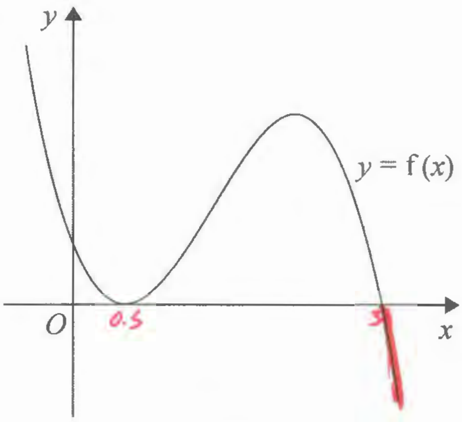

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$.

(d) Use your answer to part (c) and the sketch to deduce the set of values of x for which f(x) $\leqslant 0$ **(2)**

(a) $F(1) = -4(1)^3 + 16(1)^2 - 13(1) + 3 = 2$

(b) $F(3) = -4(3)^3 + 16(3)^2 - 13(3) + 3 = 0$ $\therefore (x-3)$ is a Factor

(c)
$$\begin{array}{r} -4x^2 + 4x - 1 \\ x-3 \overline{\big) -4x^3 + 16x^2 - 13x + 3} \\ -4x^3 + 12x^2 \\ \hline 4x^2 - 13x + 3 \\ 4x^2 - 12x \\ \hline -x + 3 \\ -x + 3 \\ \hline 0 \end{array}$$

(d) $\boxed{x \geqslant 3}$

$F(x) = (x-3)(-4x^2 + 4x - 1)$

$F(x) = -(x-3)(4x^2 - 4x + 1)$

$F(x) = -(x-3)(2x-1)^2$

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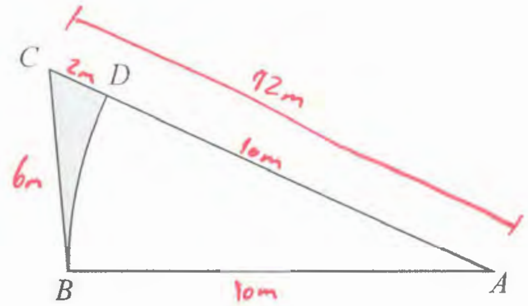

Figure 2

Figure 2 shows a sketch of a design for a triangular garden *ABC*.

The garden has sides *BA* with length 10 m, *BC* with length 6 m and *CA* with length 12 m.

The point *D* lies on *AC* such that *BD* is an arc of the circle centre *A*, radius 10 m.

A flowerbed *BCD* is shown shaded in Figure 2.

(a) Find the size of angle *BAC*, in radians, to 4 decimal places.
(2)

(b) Find the perimeter of the flowerbed *BCD*, in m, to 2 decimal places.
(3)

(c) Find the area of the flowerbed *BCD*, in m^2, to 2 decimal places.
(4)

(a) $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$\cos A = \dfrac{12^2+10^2-6^2}{2(12)(10)} = \dfrac{13}{15}$

$\angle A = \cos^{-1}\left(\dfrac{13}{15}\right) = 0.5223^{\text{rad}}$

(b) Arc $BD = r\theta^{\text{rad}} = 10(0.5223) = 5.223$ m

Perimeter $= 6 + 2 + 5.223 \simeq 13.22$ m

(c) Area of $\triangle ABC = \frac{1}{2}(10)(12)\sin(0.5223)$

$= 29.932$ m^2

Area of Sector $ABD = \frac{1}{2}(10)^2(0.5223)$

$= 26.115$ m^2

Area of $BCD = 29.932 - 26.115 \simeq 3.82$ m^2

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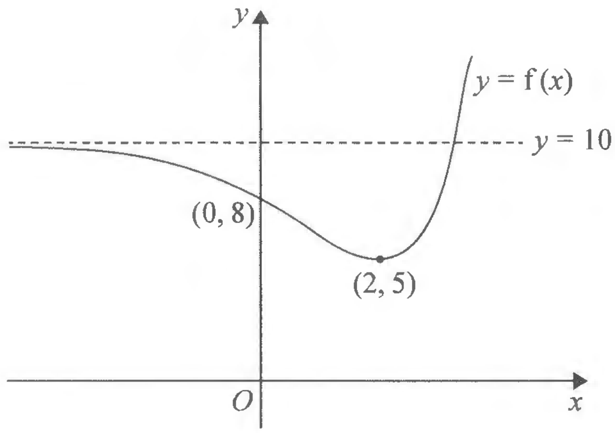

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.

The curve crosses the y-axis at the point (0, 8).

The line with equation $y = 10$ is the only asymptote to the curve.

The curve has a single turning point, a minimum point at (2, 5), as shown in Figure 3.

(a) State the coordinates of the minimum point of the curve with equation $y = \mathrm{f}\left(\frac{1}{4}x\right)$

(1)

(b) State the equation of the asymptote to the curve with equation $y = \mathrm{f}(x) - 3$

(1)

The curve with equation $y = \mathrm{f}(x)$ meets the line with equation $y = k$, where k is a constant, at two distinct points.

(c) State the set of possible values for k.

(2)

(d) Sketch the curve with equation $y = -\mathrm{f}(x)$. On your sketch, show clearly the coordinates of the turning point, the coordinates of the intersection with the y-axis and the equation of the asymptote.

(3)

(a) (8, 5)

(b) $y = 7$

(c) $5 < K < 10$

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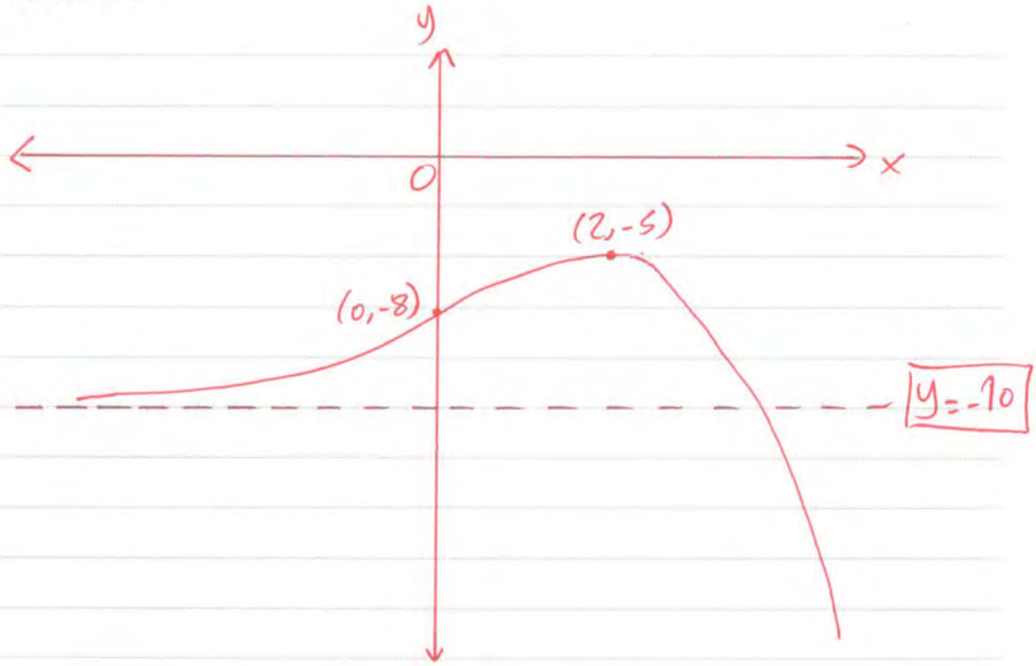
Question 7 continued



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8. (a) Find $\int(3x^2 + 4x - 15)\,dx$, simplifying each term.

(3)

Given that b is a constant and

$$\int_b^4 (3x^2 + 4x - 15)\,dx = 36$$

(b) show that $b^3 + 2b^2 - 15b = 0$

(2)

(c) Hence find the possible values of b.

(3)

(a) $\frac{3x^3}{3} + \frac{4x^2}{2} - 15x + C$

$x^3 + 2x^2 - 15x + C$

(b) $\left[x^3 + 2x^2 - 15x\right]_b^4 = 36$

$\left((4)^3 + 2(4)^2 - 15(4)\right) - \left(b^3 + 2b^2 - 15b\right) = 36$

$36 - (b^3 + 2b^2 - 15b) = 36$

$b^3 + 2b^2 - 15b = 0$ QED

(c) $b(b^2 + 2b - 15) = 0$

$b(b+5)(b-3) = 0$

$\boxed{b=0}$ $\boxed{b=-5}$ $\boxed{b=3}$

Leave blank

9. (i) Find the exact value of x for which

$$2\log_{10}(x-2) - \log_{10}(x+5) = 0$$

(5)

(ii) Given

$$\log_p(4y+1) - \log_p(2y-2) = 1 \qquad p > 2, y > 1$$

express y in terms of p.

(5)

(i) $\log_{10}(x-2)^2 - \log_{10}(x+5) = 0$

$\log_{10}\left(\frac{(x-2)^2}{x+5}\right) = 0$

$\frac{(x-2)^2}{x+5} = 10^0$

$\frac{x^2-4x+4}{x+5} = 1$

$x^2-4x+4 = x+5$

$x^2-5x-1=0$

$a=1 \quad b=-5 \quad c=-1$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{(-5)^2-4(1)(-1)}}{2(1)} = \frac{5 \pm \sqrt{29}}{2}$

$\frac{5-\sqrt{29}}{2}$ is rejected since it is negative

$\boxed{x = \frac{5+\sqrt{29}}{2}}$

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Question 9 continued

(ii) $\log_p\left(\frac{4y+1}{2y-2}\right) = 1$

$\frac{4y+1}{2y-2} = p$

$4y+1 = 2py - 2p$

$2p+1 = 2py - 4y$

$2p+1 = y(2p-4)$

$\boxed{y = \frac{2p+1}{2p-4}}$

Q9

(Total 10 marks)

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10. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{8}\right)^{10}$$

giving each term in its simplest form.

(4)

$$f(x) = \left(2 - \frac{x}{8}\right)^{10}(a + bx), \text{ where } a \text{ and } b \text{ are constants}$$

Given that the first two terms, in ascending powers of x in the series expansion of $f(x)$, are 256 and $352x$,

(b) find the value of a,

(2)

(c) find the value of b.

(2)

(a) 1st term $\quad {}^{10}C_0 . (2)^{10} . \left(-\frac{x}{8}\right)^0 = 1 . 1024 . 1 = 1024$

2nd term $\quad {}^{10}C_1 . (2)^9 . \left(-\frac{x}{8}\right)^1 = 10 . 512 . \left(-\frac{x}{8}\right) = -640x$

3rd term $\quad {}^{10}C_2 . (2)^8 . \left(\frac{x}{8}\right)^2 = 45 . 256 . \left(\frac{x^2}{64}\right) = 180x^2$

$\left(2 - \frac{x}{8}\right)^{10} = 1024 - 640x + 180x^2 + \ldots\ldots$

(b) $(1024 - 640x)(a + bx) = 1024a + 1024bx - 640ax - 640bx^2$

Ignored

$1024a = 256$

$\boxed{a = \frac{1}{4}}$

(c) $1024b - 640a = 352$

$1024b - 640\left(\frac{1}{4}\right) = 352$

$1024b = 512$

$\boxed{b = \frac{1}{2}}$

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11. Wheat is to be grown on a farm.

A model predicts that the mass of wheat harvested on the farm will increase by 1.5% per year, so that the mass of wheat harvested each year forms a geometric sequence.

Given that the mass of wheat harvested during year one is 6000 tonnes,

(a) show that, according to the model, the mass of wheat harvested on the farm during year 4 will be approximately 6274 tonnes.
(2)

During year N, according to the model, there is predicted to be more than 8000 tonnes of wheat harvested on the farm.

(b) Find the smallest possible value of N.
(5)

It costs £5 per tonne to harvest the wheat.

(c) Assuming the model, find the total amount that it would cost to harvest the wheat from year one to year 10 inclusive. Give your answer to the nearest £1000.
(3)

(a) $a = 6000 \qquad r = 1.015$

$U_4 = ar^3$
$= 6000(1.015)^3 \simeq 6274$ tonnes QED

(b) $U_n > 8000$
$6000(1.015)^{n-1} > 8000 \quad (\div 6000)$
$1.015^{n-1} > \frac{4}{3}$
$\log 1.015^{n-1} > \log \frac{4}{3}$
$(n-1)\log 1.015 > \log \frac{4}{3}$
$n-1 > \frac{\log \frac{4}{3}}{\log 1.015}$
$n > 1 + \frac{\log \frac{4}{3}}{\log 1.015}$
$n > 20.3$ $\qquad \boxed{\therefore N = 21}$

(c) $S_{10} = \frac{a(1-r^n)}{1-r}$
$= \frac{6000(1-1.015^{10})}{1-1.015} \simeq 64216$ tonnes

Total cost $= 5 \times 64216 = 321080 \simeq \boxed{£321000}$

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12.

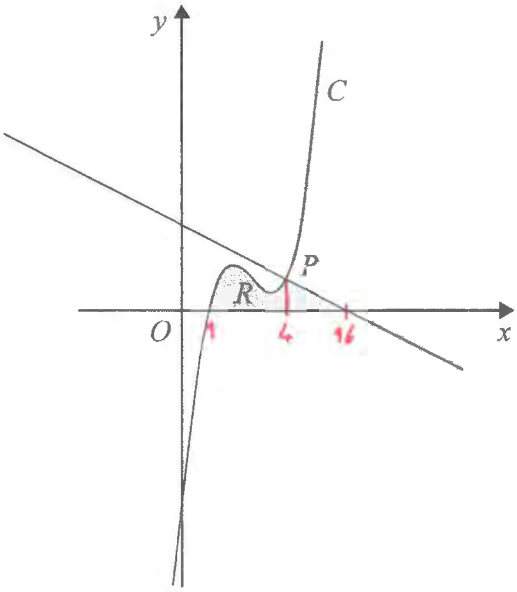

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = x^3 - 9x^2 + 26x - 18$$

The point P (4, 6) lies on C.

(a) Use calculus to show that the normal to C at the point P has equation

$$2y + x = 16$$

(5)

The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the normal to C at P.

(b) Show that C cuts the x-axis at (1, 0)

(1)

(c) Showing all your working, use calculus to find the exact area of R.

(6)

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(a) $\frac{dy}{dx} = 3x^2 - 18x + 26$

At $x=4$ $\frac{dy}{dx} = 3(4)^2 - 18(4) + 26 = 2 \rightarrow$ m of tangent

$m = -\frac{1}{2}$ $(4,6)$

$y = -\frac{1}{2}x + c$

$6 = -\frac{1}{2}(4) + c$

$c = 8$

$y = -\frac{1}{2}x + 8 \xrightarrow{(\times 2)} \boxed{2y + x = 16}$ ~~XX~~ QED

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Question 12 continued

(b) $y = (1)^3 - 9(1)^2 + 26(1) - 18 = 0$

$\therefore$ Point $(1,0)$ lies on C

(c) $\int_1^4 (x^3 - 9x^2 + 26x - 18)\,dx$

$\left[\frac{x^4}{4} - \frac{9x^3}{3} + \frac{26x^2}{2} - 18x\right]_1^4$

$\left[\frac{x^4}{4} - 3x^3 + 13x^2 - 18x\right]_1^4$

$\left(\frac{(4)^4}{4} - 3(4)^3 + 13(4)^2 - 18(4)\right) - \left(\frac{1}{4} - 3 + 13 - 18\right)$

$(8) \quad - \left(-\frac{31}{4}\right) \quad = 15.75 \text{ units}^2$

$2y + x = 16$

At $y = 0 \quad x = 16$

Area of $\triangle = \frac{1}{2} \times 6 \times 12 = 36 \text{ units}^2$

Area of $R = 15.75 + 36 = \boxed{51.75 \text{ units}^2}$

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13. (a) Show that the equation

$$5\cos x + 1 = \sin x \tan x$$

can be written in the form

$$6\cos^2 x + \cos x - 1 = 0$$

(4)

(b) Hence solve, for $0 \leqslant \theta < 180°$

$$5\cos 2\theta + 1 = \sin 2\theta \tan 2\theta$$

giving your answers, where appropriate, to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(a) $5\cos x + 1 = \sin x \cdot \frac{\sin x}{\cos x}$

$5\cos x + 1 = \frac{\sin^2 x}{\cos x}$ $(\times \cos x)$

$5\cos^2 x + \cos x = \sin^2 x$

$5\cos^2 x + \cos x = 1 - \cos^2 x$

$6\cos^2 x + \cos x - 1 = 0$ QED

(b) Let 2θ be x

$0 \leqslant \theta < 180$

$0 \leqslant x < 360$

$6\cos^2 x + \cos x - 1 = 0$

$(3\cos x - 1)(2\cos x + 1) = 0$

$\cos x = \frac{1}{3}$ | $\cos x = -\frac{1}{2}$

1st Quad $x = 70.53°$ | 2nd Quad $x = 120°$

4th Quad $x = 289.47°$ | 3rd Quad $x = 240°$

$\theta = \{35.3°, 144.7°, 60°, 120°\}$

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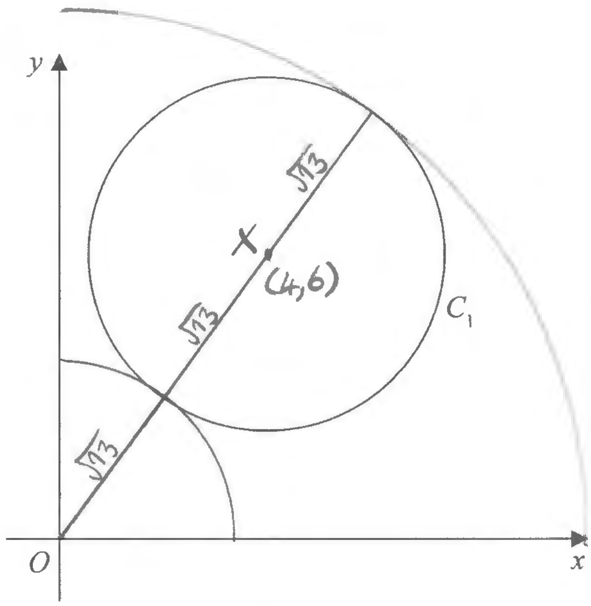

Figure 5

Figure 5 shows a sketch of the circle C_1

The points $A(1, 4)$ and $B(7, 8)$ lie on C_1

Given that AB is a diameter of the circle C_1

(a) find the coordinates for the centre of C_1

(2)

(b) find the exact radius of C_1 simplifying your answer.

(2)

Two distinct circles C_2 and C_3 each have centre (0, 0).

Given that each of these circles touch circle C_1

(c) find the equation of circle C_2 and the equation of circle C_3

(4)

(a) $\left(\frac{1+7}{2}, \frac{4+8}{2}\right) = (4,6)$

(b) $AB = \sqrt{(8-4)^2+(7-1)^2} = 2\sqrt{13}$ $\therefore$ Radius $= \sqrt{13}$

(c) $OX = \sqrt{4^2+6^2} = 2\sqrt{13}$ $\therefore$ Radius of $C_2 = \sqrt{13}$

Eq. of $C_2 \Rightarrow x^2+y^2 = 13$

Eq. of $C_3 \Rightarrow x^2+y^2 = 117$

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15. The height of water, H metres, in a harbour on a particular day is given by the equation

$$H = 4 + 1.5\sin\left(\frac{\pi t}{6}\right), \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight, and $\frac{\pi t}{6}$ is measured in radians.

(a) Show that the height of the water at 1 a.m. is 4.75 metres. **(1)**

(b) Find the height of the water at 2 p.m. **(2)**

(c) Find, to the nearest minute, the first two times when the height of the water is 3 metres.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*) **(6)**

(a) $H = 4 + 1.5\sin\left(\frac{\pi(1)}{6}\right) = 4.75\text{m}$ QED

(b) $H = 4 + 1.5\sin\left(\frac{\pi(14)}{6}\right) = 5.30\text{m}$

(c) let $\frac{\pi t}{6}$ be θ $\quad 3 = 4 + 1.5\sin\theta \quad$ $0 \leqslant t < 24$

$\sin\theta = \frac{-1}{1.5} \quad$ $0 \leqslant \theta < 4\pi$

3rd Quad $\theta = 3.8713^{\text{rad}} \rightarrow t = 7.394$ hours $\rightarrow$ 7:24 am

4th Quad $\theta = 5.5535^{\text{rad}} \rightarrow t = 10.606$ hours $\rightarrow$ 10:36 am

$\therefore$ First two times are 7:24am & 10:36am

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